Statistical Illusion – Friendship Paradox

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Topic we covered

- Monty hall problem
- Friendship paradox
- Simpson's paradox
- Waiting time paradox
- Certainty and possibility effects
- Limitations in common summary statistics

Question

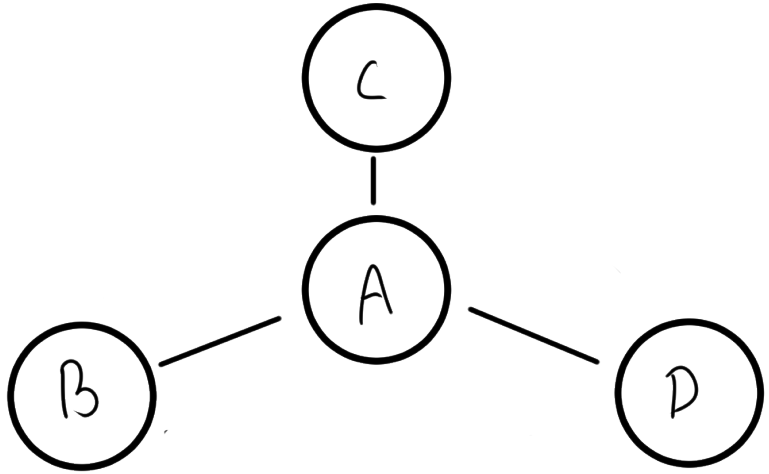
Do you think you have more, less or same number of friends as your friends on average?

Friendship Paradox

- The paradox that most people have less friends than their friends on average
- Can also be applied to any social networks
  - e.g. your twitter followers will have more followers than you do on average
  - e.g. your partner will have more partners in the past than you do on average
  - e.g. predicting infection in social networks

Examples

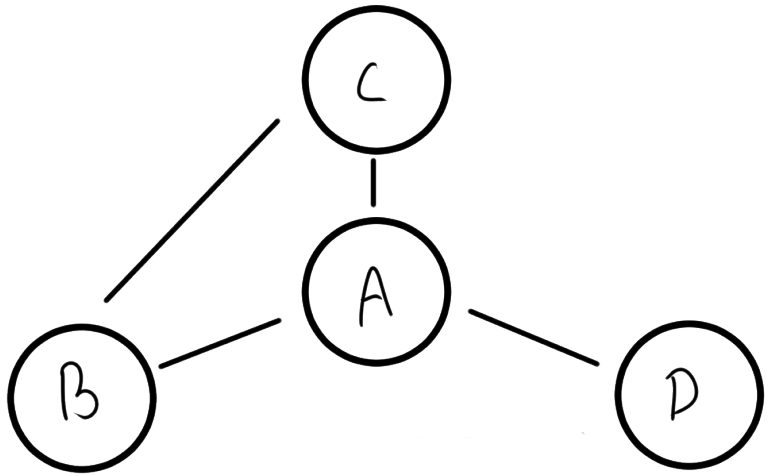
- Number of friends A has: 3
  - Number of friends A's friends have:
    - B: 1
    - C: 1
    - D: 1
    - Total: 3
- Number of friends B has: 1
  - Number of friends B's friends have:
    - A: 3
    - Total: 3
- Number of friends C has: 1
  - Number of friends C's friends have:
    - A: 3
    - Total: 3
- Number of friends D has: 1
  - Number of friends D's friends have:
    - A: 3
    - Total: 3



Mean number of friends each person has:
(3+1+1+1)/4 = 1.5
Mean number of friends each person has:
(3+3+3+3)/4 = 3

Examples

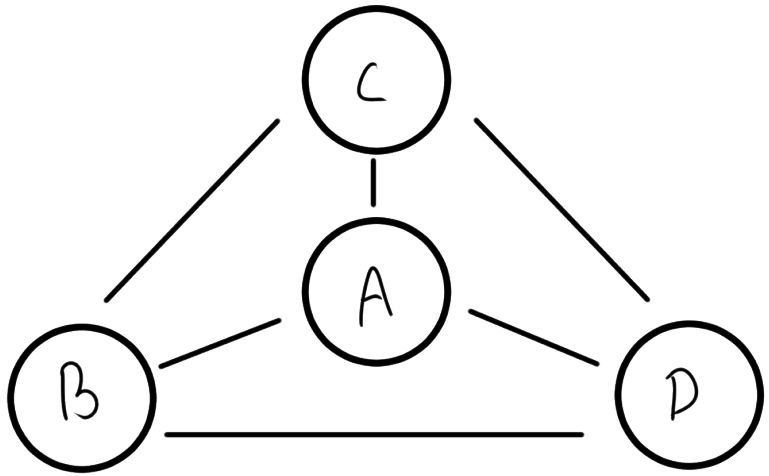
- Number of friends A has: 3
  - Number of friends A's friends have:
    - B: 2
    - C: 2
    - D: 1
    - Total: 5
- Number of friends B has: 2
  - Number of friends B's friends have:
    - A: 3
    - C: 2
    - Total: 5
- Number of friends C has: 2
  - Number of friends C's friends have:
    - A: 3
    - B: 2
    - Total: 5
- Number of friends D has: 1
  - Number of friends D's friends have:
    - A: 3
    - Total: 3



Mean number of friends each person has:
(3+2+2+1)/4 = 2
Mean number of friends each person has:
(5+5+5+3)/4 = 4.5

Examples

- Number of friends A has: 3
  - Number of friends A's friends have:
    - B: 3
    - C: 3
    - D: 3
    - Total: 9
- Number of friends B has: 3
  - Number of friends B's friends have:
    - A: 3
    - C: 3
    - D: 3
    - Total: 9
- Number of friends C has: 3
  - Number of friends C's friends have:
    - A: 3
    - B: 3
    - D: 3
    - Total: 9
- Number of friends D has: 3
  - Number of friends D's friends have:
    - A: 3
    - B: 3
    - C: 3
    - Total: 9



Mean number of friends each person has:
(3+3+3+3)/4 = 3
Mean number of friends each person has:
(9+9+9+9)/4 = 9

Conclusion

n individuals, x_i ties

Mean number of friends:

$$\frac{\Sigma(x_i)}{n}$$

Mean number of friends' friends:

$$\frac{\Sigma({x_i}^2)}{\Sigma(x_i)}$$

$$\sigma^2 = \frac{\Sigma({x_i}^2)}{n} - \mu^2$$

$$\frac{\Sigma({x_i}^2)}{n} = \sigma^2 + \mu^2$$

$$\Sigma({x_i}^2) = (\sigma^2 + \mu^2)n$$

Divide each side by $\Sigma(x_i) = \mu n$

$$\frac{\Sigma({x_i}^2)}{\Sigma(x_i)} = \frac{(\sigma^2+\mu^2)n}{\mu n} = \mu + \frac{\sigma^2}{\mu}$$

Conclusion

- $\mu = \frac{\Sigma(x_i)}{n}, \frac{\Sigma(x_i{}^2)}{\Sigma(x_i)} = \mu + \frac{\sigma^2}{\mu}$
- $\mu \leq \mu + \frac{\sigma^2}{\mu}$
- Mean number of friends is always equal to or less than mean number of friend's friends
- Mean among friends increases as the variance among individuals increases for a fixed mean number of individual's friends.

Conclusion

- Most people are likely to be within their own friends group. i.e. It is unlikely for a person to be a friend with the one with few friends.
- Friendship is disproportionate: few people have large number of friendship with others, and the others with few.
- For friends' friends, some individuals are counted more than once. The number of friends only includes each individuals once
- Using mean friends' friends number is unfair basis for judging if one has enough friends.
- Other similar paradox: class size paradox

Resources

- Friendship paradox. (2020, December 05). Retrieved December 09, 2020, from https://en.wikipedia.org/wiki/Friendship_paradox
- Feld, S. (1991). Why Your Friends Have More Friends Than You Do. *American Journal of Sociology, 96*(6), 1464-1477. Retrieved December 9, 2020, from http://www.jstor.org/stable/2781907