# Bayesian Perspective on Probability & Statistics

*An Introduction and Applied Analysis*


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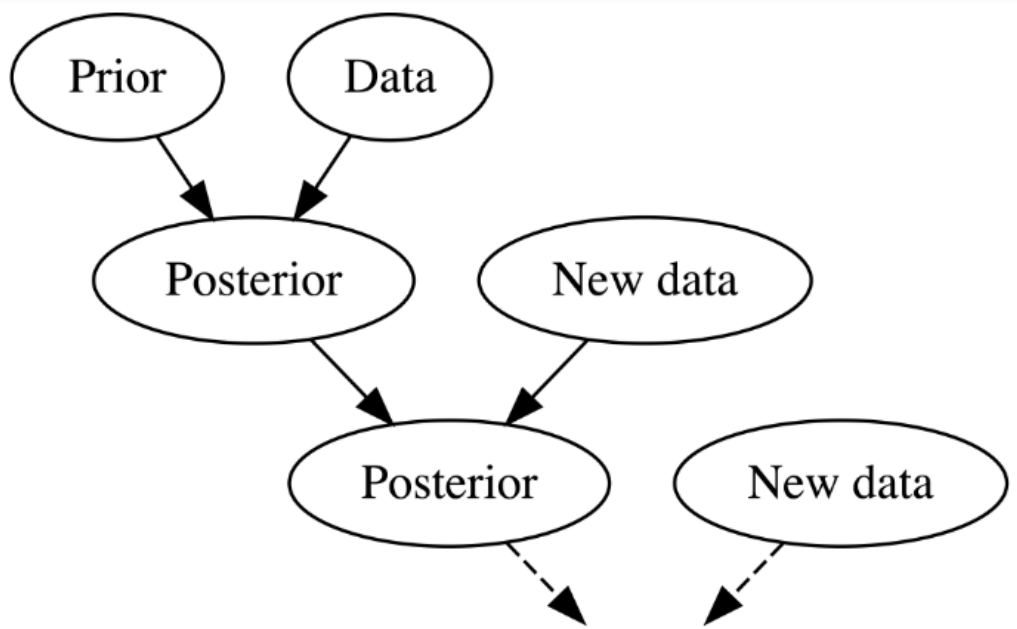
# An Intro to Bayes

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Bayesian statistics utilizes a *prior* probability with the current data to inform and create a *posterior* understanding.

**Bayes' Rule** for events *A* and *B* gives some posterior probability of *B* given *A* - can be described as (**prior** x **likelihood**) / normalizing constant.

# Advantages of Bayesian Perspective




- Builds on *prior* understanding to make more informed conclusions – and posteriors can be used as priors when more data is collected
- Can help remove noise from data…

# The Bayesian vs The Frequentist

**Bayesian Philosophy**

- Measures relative plausibility of an event
- Assesses uncertainty of hypothesis considering *observed data*
- Results are interpretable

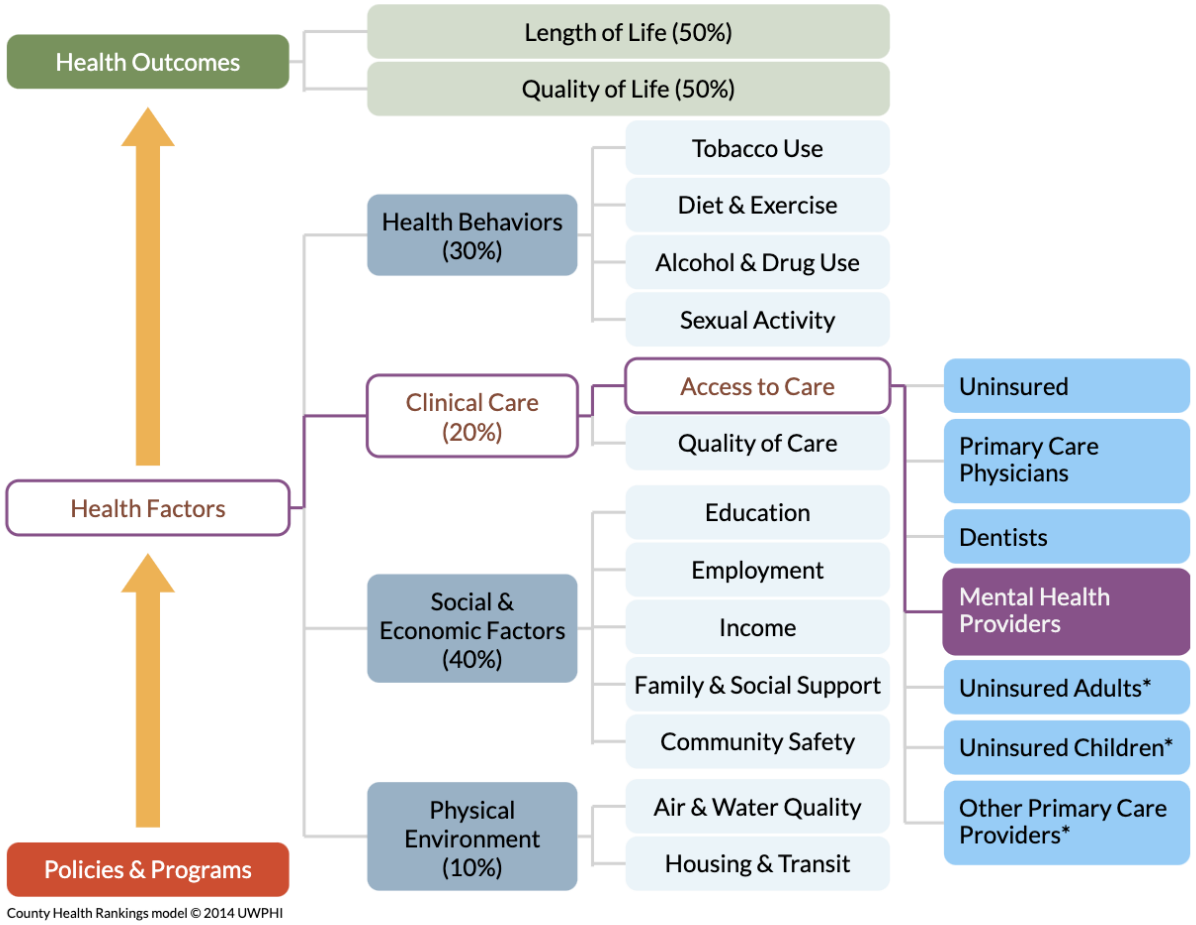
**Frequentist Philosophy**

- Measures the long-run relative frequency of a repeatable event
- Assesses uncertainty of observed data considering *assumed hypothesis*
- Results can be hard to interpret (i.e. *p-value)*

# An Application of Bayes'

Modeling the rate of mental health providers per 100k people (from population)

Data is from County Health Rankings & Roadmaps (CHR&R) - covers the number of mental health providers per county in WA (circa 2022).



# An Application of Bayes' *cont.*

**The Gamma-Poisson Model**

$$X_i|\lambda_i \sim \text{Pois}(\frac{N_i}{c}\lambda_i)$$

$$\lambda_i \sim \text{Gamma}(\alpha, \beta)$$

The prior is from the mean and variance of the mental health provider rates from all counties

$$E(\lambda_i) = \frac{\alpha}{\beta}$$

$$\text{Var}(\lambda_i) = \frac{\alpha}{\beta^2}$$

In which we found $\alpha = 6.165$ and $\beta = 0.019$.

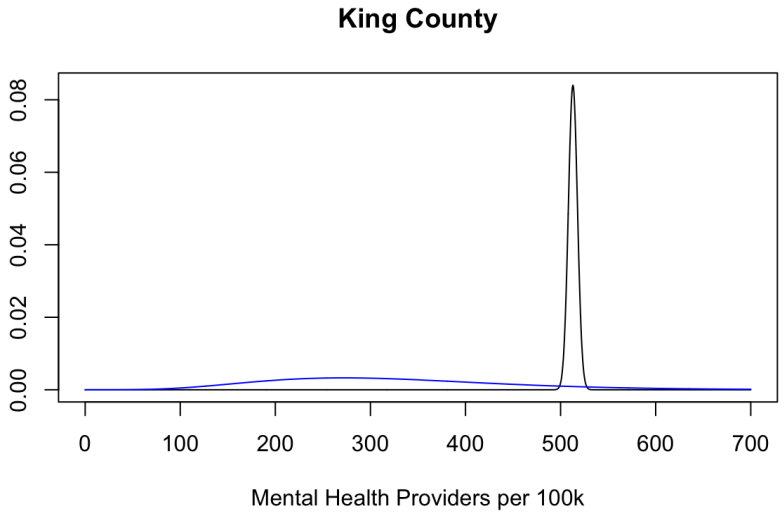
Using the prior and the data gathered from each county, the posterior is given by

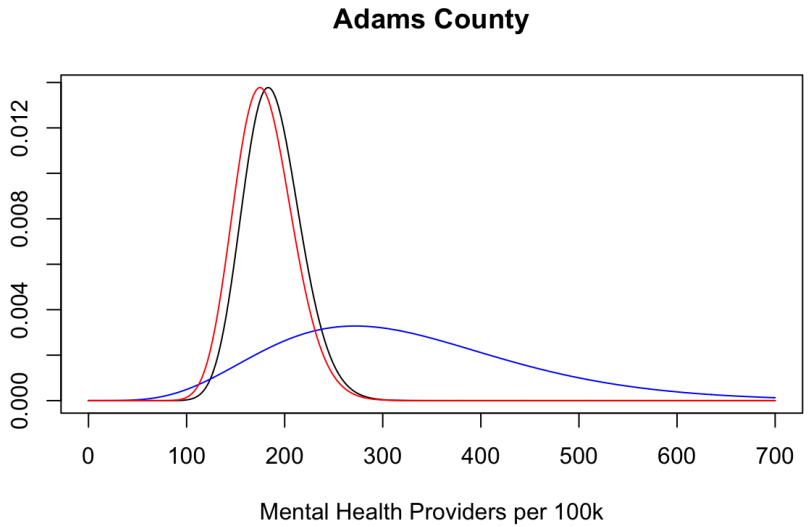
$$\lambda_i|X_i \sim \text{Gamma}(\alpha + X_i, \beta + \frac{N_i}{c})$$

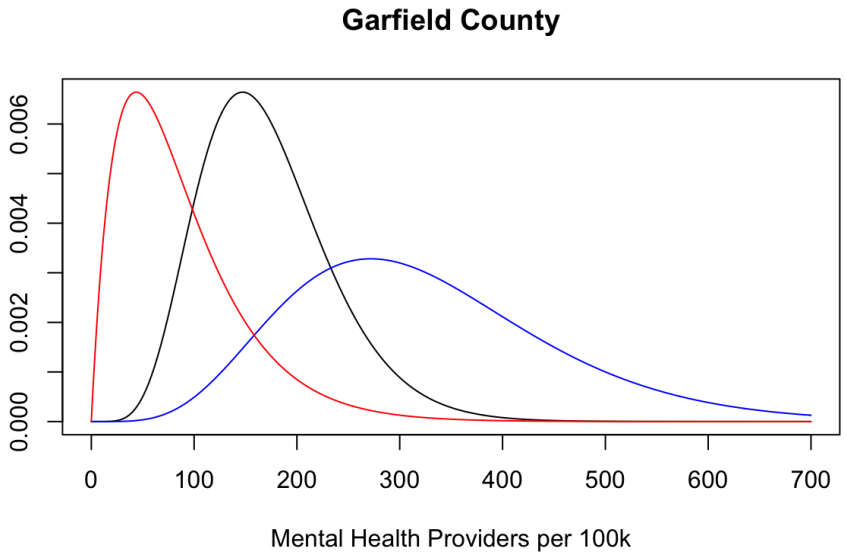
Where the adjusted rate is

$$E(\lambda_i|X_i) = \frac{\alpha + X_i}{\beta + \frac{N_i}{c}}$$

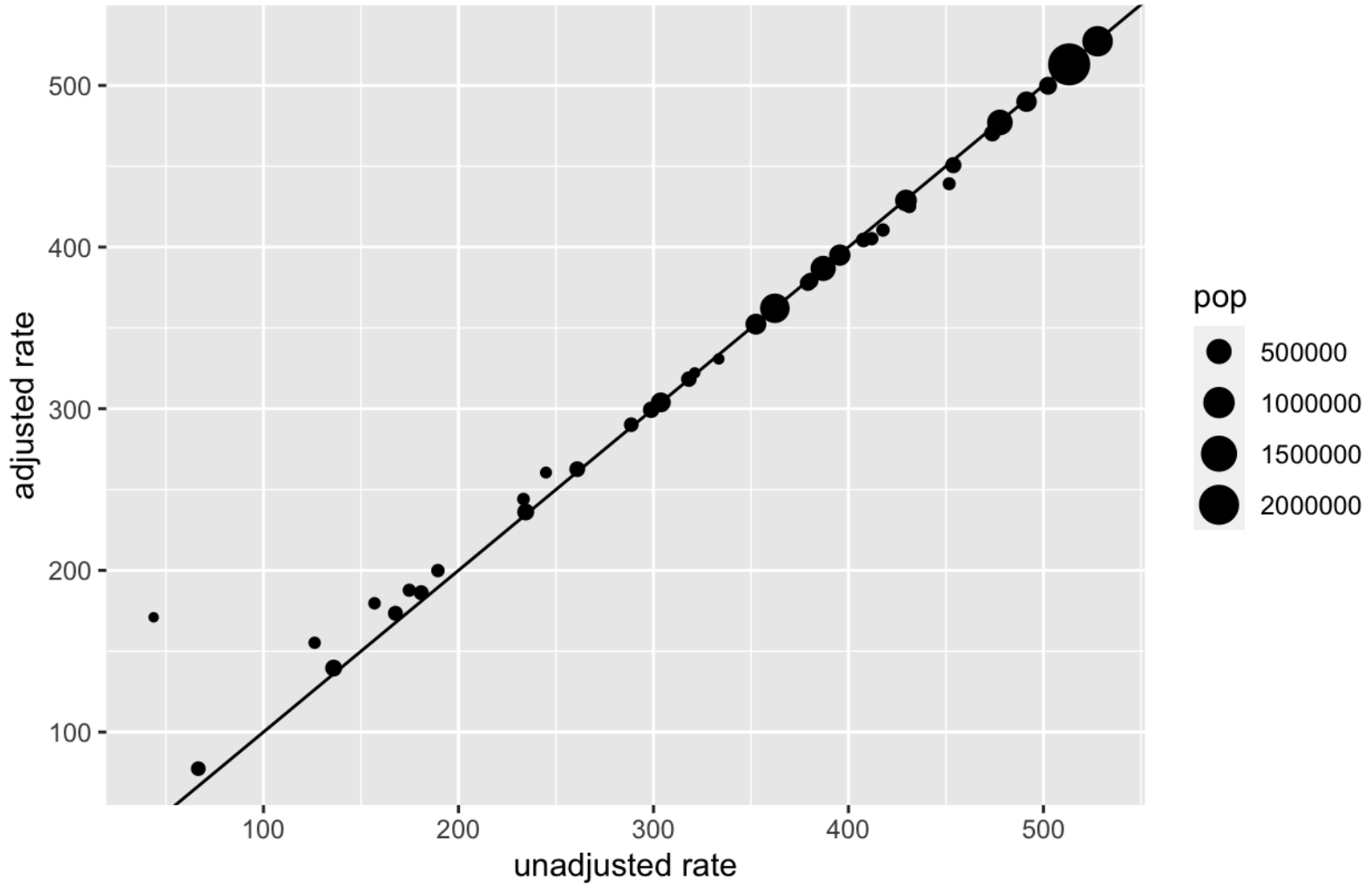
# An Application of Bayes' *cont.*



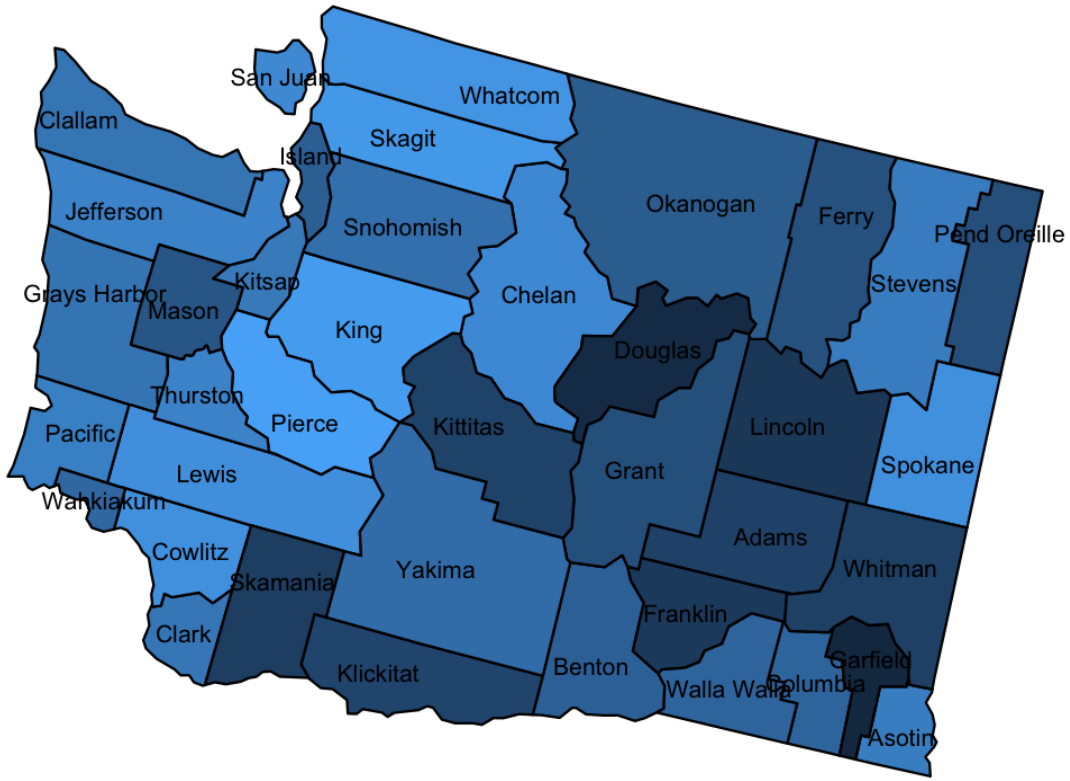



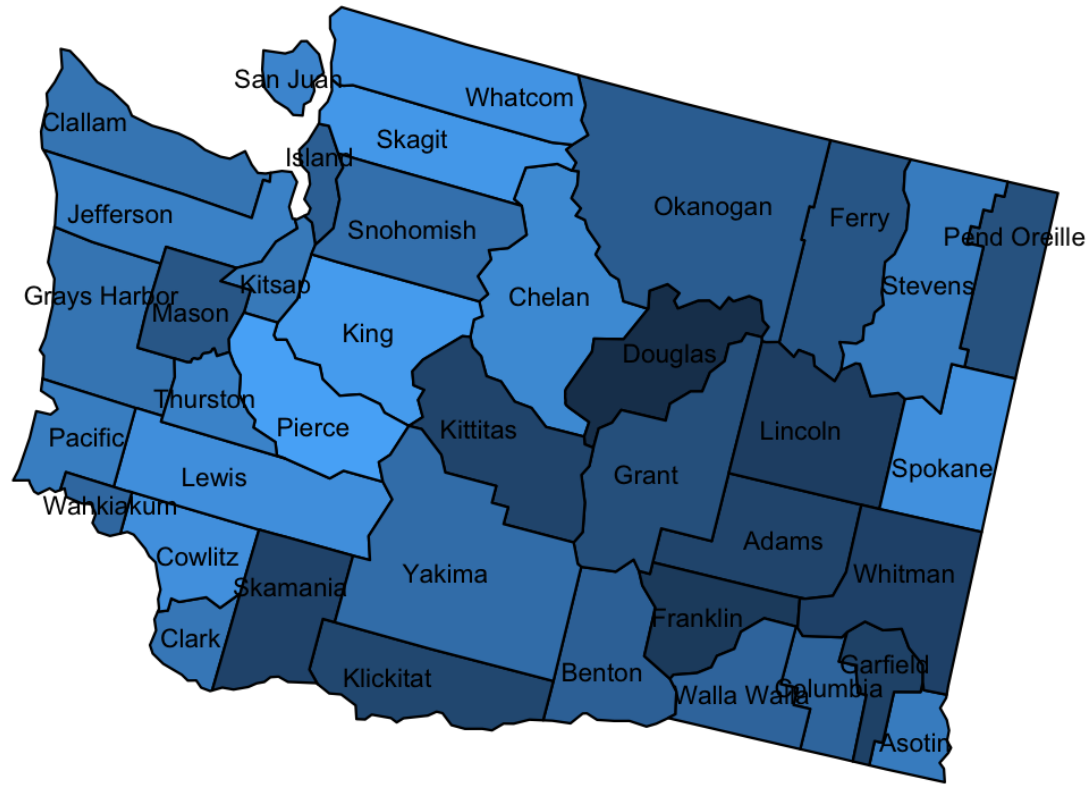

Likelihood
Prior
Posterior
(Adjusted Rate)

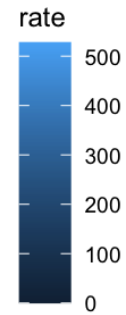

adjusted rate
500
400
300
200
100
100
200
300
400
500
unadjusted rate
pop
500000
1000000
1500000
2000000

Unadjusted Rate per 100k




Adjusted Rate per 100k



# Thank you for listening!

*Any questions?*