

ETHAN ANCELL & ABIGAIL CUMMINGS

*TOPICS IN  
RANDOM  
MATRIX THEORY*



The background features several thin, purple lines that intersect to form various geometric shapes, including triangles and quadrilaterals, creating a modern, abstract pattern.

*FREE  
PROBABILITY*

# *FREE PROBABILITY*

- Branch of mathematics that deals with random variables that are free from classical probability constraints
- Understanding the behavior of non-commutative random variables
- Free independence: captures the idea that certain non-commutative random variables can behave as if they were independent
- The application of the Central Limit Theorem (CLT)

# *FREE PROBABILITY IN RMT*

- Understanding the behavior of large matrices with random entries
- Non-commutativity
  - Random matrices A and B when multiplied:
    - $AB \neq BA$
  - Important tool used to study random matrices
- Asymptotic Behavior
  - RMT concerned with the behavior of matrices as their size grows large
- Applications
  - Random matrix theory
  - Quantum information theory
  - Integral Systems

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# *CONVERGENCES*

# *CONVERGENCE OF RANDOM VARIABLES*

- Suppose we have the sequence of random variables  $X_1, X_2, X_3, \dots$
- What does it mean for the sequence to converge to some  $X$ ?

# *CONVERGENCE OF A SEQUENCE OF REAL NUMBERS*

- Describes the behavior of the sequence as it progresses towards a specific value or limit
- $\lim_{n \rightarrow \infty} a_n = L$  or  $\forall \varepsilon > 0, \exists N$  such that  $\forall n \geq N, |a_n - L| < \varepsilon$

# *CONVERGENCE IN PROBABILITY*

- Fundamental concept in statistics and probability theory, providing a way to describe the behavior of random variables as sample sizes grow large
  - In other words: the likelihood of the random variables being close to  $c$  become increasingly high as the number of observations increases indefinitely
- $\lim_{n \rightarrow \infty} P(|X_n - c| \geq \varepsilon) = 0$
- Forming the basis for many statistical inference models