In mathematics, particularly in the domain of random matrices, lies intricate topics to be explored. This essay will reflect the complexity of the research done in the winter Directed Reading Program (DRP) of 2024 by explaining some of the high level topics seen in Random Matrix Theory (RMT) including free probability, its application in RMT and the understanding of convergences.

The concept of free probability is under the umbrella of classical probability theory, but contrarily has more random variables that are not bound by commutativity which gives us a better understanding of randomness. This branch of mathematics is primarily focused on the study of non-commutative random variables. These variables do not follow the conventional rules of probability, yet their behavior is not devoid of order. Free Probability introduces the notion of free independence, a concept where certain non-commutative random variable exhibit independence-like behavior despite their interdependence. This shift enables us to comprehend the behavior of sums of non-commutative random variables, giving us an insight into the distributions of random matrices.

Within the realm of Random Matrix Theory, Free Probability helps to understand this concept. The non-commutativity inherent in matrices creates a challenge that can be addressed by using Free Probability. When inspecting matrices with random entries, Free Probability provides a framework to navigate non-commutativity. With this, we discern the asymptotic behavior of the these matrices by investigating the eigenvalues and eigenvectors. Such behaviors and findings can also be observed outside mathematics including finding applications in physics, integral systems, and the study of phase transitions.

While working with Random Matrix Theory, the concept of convergences helps us understand the limits of sequences of matrices. Just as convergence in probability has the ability to “explain” the behavior of random variables as samples sizes grow large, convergence in the context of matrices offers an explanation of the order found in chaos. With this, sequences of matrices approach their limit. With convergence, we are able to see the gradual alignment of sequences as they approach a single point and can see where randomness converges into a discernible pattern.

In the latter half of the DRP, we found importance in the focus of Free Probability and what it offers to Random Matrix Theory, the application of Free Probability, and the study of convergences. These concepts highlighted the order and what seems to be a complex study of random matrices.